\[ d(f, f^*)^2 = \inf \int_D \|x - y\|^2 \, d\mu(y) \quad \text{if} \quad f, f^* \text{ are probability densities} \]

\[ d(f, f^*) < 0 \quad \Rightarrow \quad \exists \text{ unique map} \]

\[ \phi : D \to D \]

\[ \int_D \int_D \|x - y\|^2 \, d\mu(x) \, d\mu(y) = \int_D \int_D \zeta(\phi(x)) \, f^*(\phi(x)) \, d\mu(x) \, d\mu(y) \quad \forall \zeta \in C(C(D)) \]

\[ \phi(x) = F^{-1}(F(x)) \text{, } x \in D, \]

\[ F(x) = \int_0^x f(x') \, dx', \quad F^*(x) = \int_0^x f^*(x') \, dx' \]

\[ \frac{1}{c} d(f, f^*)^2 = \int_D \|x - \phi(x)\|^2 \, f^* \, dx \]

\[ \frac{1}{c} d(f, f^*)^2 = \inf \int_C \int_D \sigma^2 f \, d\mu \, d\nu \quad \text{can show by Schwarz inequality} \]

One deformation path \( f(\xi, \tau) \) subject to \( f_t + \left(\frac{\partial f}{\partial \xi}\right)_\xi = 0 \) (continuity)

\[ f(\xi, 0) = f^*(\xi), \quad f(\xi, 1) = f(\xi) \text{ (well defined)} \]

\[ \left\{ \begin{array}{l}
\frac{\partial f}{\partial \tau} + \nabla f \cdot \nabla \xi = 0 \quad \text{(Poisson)} \quad \text{for extremal pair} (f, \sigma) \\
\frac{\partial}{\partial \alpha} d(f, f^*)^2 = 0 \quad \text{on extremal} \\
\end{array} \right. \]

\[ \mu \int_0^\tau \int_C \left( \frac{\partial f}{\partial t} \right)^2 \, d\alpha \, dt \quad \rightarrow \quad \int_0^\tau \int_C \sigma^2 f \, d\nu \, d\tau \]

\[ f_t + \left(\frac{\partial f}{\partial \xi}\right)_\xi = 0 \]

\[ f(x, 0) = 0, \quad f(x, T) = f(x) \]

\[ \frac{\partial}{\partial \alpha} f(x, T) = 0 \]
make an estimate
\[ \Omega = (0, 1), \quad \rho(\alpha, \xi) \geq \delta > 0 \quad \text{in} \: \Omega, \: t > 0 \]
\[
\frac{1}{t} \int_0^t \int_\Omega \rho \partial_\xi \xi \, dx \, dt \leq \frac{C_A}{\min \rho} \int_0^t \int_\Omega \rho^2 \, dx \, dt
\]
\[ \rho^* = \rho|_{t=0}, \: \delta = \rho|_{t=0} \]

\((\eta, \rho)\) meets weak continuous equality: \(0 < t < 2\)

\[
\frac{\partial F}{\partial t} + \rho \frac{\partial F}{\partial x} = \frac{\partial F}{\partial t} + \rho \frac{\partial F}{\partial t} = 0 \quad \text{in} \: \Omega
\]

\[ F \bigg|_{t=0} = 0, \quad \rho \bigg|_{t=0} = 0 \quad \Rightarrow \quad \rho = 0 \]

\[
\rho \bigg|_{t=0} = 0, \quad \rho \bigg|_{t=0} = 0
\]

\[
\left( \rho \bigg|_{t=0} \right) \text{ meets weak continuous equality:} \quad 0 < t < 2
\]

\[ \frac{\partial F}{\partial t} + \rho \frac{\partial F}{\partial x} = \frac{\partial F}{\partial t} + \rho \frac{\partial F}{\partial t} = 0 \quad \text{in} \: \Omega
\]

\[ 0 < t < 2 \]

\[ \int_0^t \int_\Omega \rho \partial_\xi \xi \, dx \, dt \leq \int_0^t \int_\Omega \frac{\rho(\xi)}{\min \rho} \, dx \, dt \int_\Omega \rho(\xi) \, dy \, dt
\]

\[ \Rightarrow \quad \rho \bigg|_{t=0} = 0, \quad \rho \bigg|_{t=0} = 0
\]

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\]

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\]

\[
\Rightarrow \quad \rho \bigg|_{t=0} = 0, \quad \rho \bigg|_{t=0} = 0
\]

\[ \text{assume now that our path is the most likely to occur}
\]

\[ \frac{1}{2t} d(\rho, \rho^*) \xi + F_2(\rho) \bigg|_{t=2} \leq \frac{1}{2t} d(\rho, \rho^*) \xi + F_2(\rho) \bigg|_{t=2}
\]

\[ \Rightarrow \quad \text{for each velocity line, have} \: \rho \bigg|_{t=0} = 0
\]
\[ \Rightarrow \quad u \rightarrow e \]

\[ p \frac{\partial \rho}{\partial t} = \frac{1}{\partial \alpha} \left( \frac{2}{\partial \alpha} + \psi \rho \right) \quad \text{Fokker-Planck} \]

\[ \lambda \approx ? \quad \text{entropy method} \]