x area

t time

$f_n(x,t)$ density of n-sided grains with area x at time t

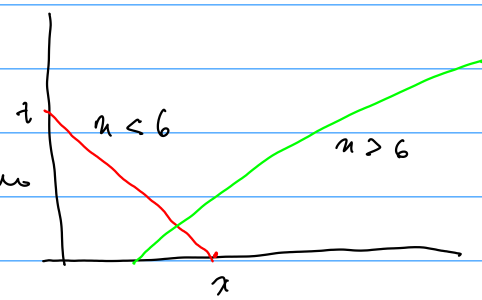
$$\frac{\partial f_n}{\partial t} + \alpha(n-6)\frac{\partial f_n}{\partial x} = A_{n\,n-1} f_{n-1} + A_{nn} f_n + A_{n\,n+1} f_{n+1} \qquad x > 0,\ t > 0,$$

$$3 = n_{min} \leqq n \leqq n_{max} \sim 12 - 14$$

$f_n(0,t) = 0 \quad t > 0,\ n \geqq 6$

$f_n(X,t) = 0 \quad t > 0,\ n < 6$

X = area of configuration



$\sum_i A_{in} = 0 \qquad A_{n-1} \geqq 0 \qquad A_n \leqq 0$

$\qquad A_{n\,n+1} \geqq 0$

noticed a degeneracy: $\quad f_n(x,t_0) = 0 \implies$ all $f_n = 0$ for $t \geqq t_0$

solution extinguishes

$P = I + \tau A, \quad A = (A_{ij})$

prob matrix

constraints:

① $\frac{d}{dt}\sum (n-6)\int_0^\infty f_n\,dx = 0$ $\qquad$ Euler $\qquad$ C S Smith

$\sum (n-6)\int_0^\infty f_n(x,0)\,dx = 0$

② $\sum \int_0^\infty x f_n\,dx = \text{constant}$ $\qquad$ fixed area

$$0 = \frac{d}{dt}\sum (n-6)\int_0^\infty f_n\,dx = \sum (n-6)\int_0^\infty \frac{\partial f_n}{\partial t}\,dx$$

$$= -\alpha \sum (n-6)^2 \int_0^\infty \frac{\partial f_n}{\partial x}\,dx + \sum (n-6)\int_0^\infty A_{nj} f_j\,dx$$

$$= \alpha \sum_3^5 (n-6)^2 f_n(0,t) + \sum (n-6)\int_0^\infty A_{nj} f_j\,dx$$

① $$\alpha \sum_3^5 (n-6)^2 f_n(0,t) = -\sum_3^{n_{max}} (n-6)\int_0^\infty A_{nj} f_j\,dx$$

$$0 = \frac{d}{dt}\sum\int_0^\infty x f_n\,dx = \sum\int_0^\infty x\frac{\partial f_n}{\partial t}\,dx$$

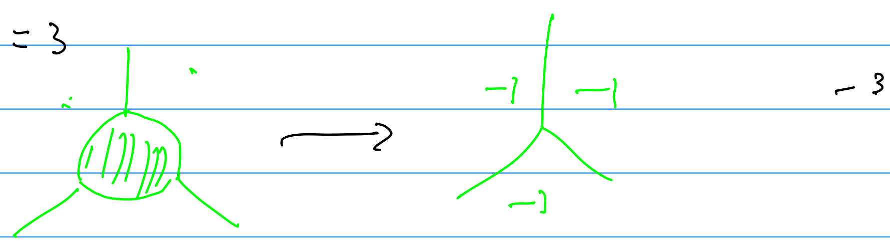
$$= -\alpha\sum(n-6)\int_0^\infty x\frac{\partial f_n}{\partial x}\,dx + \underbrace{\sum\int_0^\infty x A_{nj} f_j\,dx}_{=0}$$

$$= \alpha\sum\underbrace{(n-6)\int_0^\infty f_n\,dx}_{=0 \text{ by Euler}} +$$

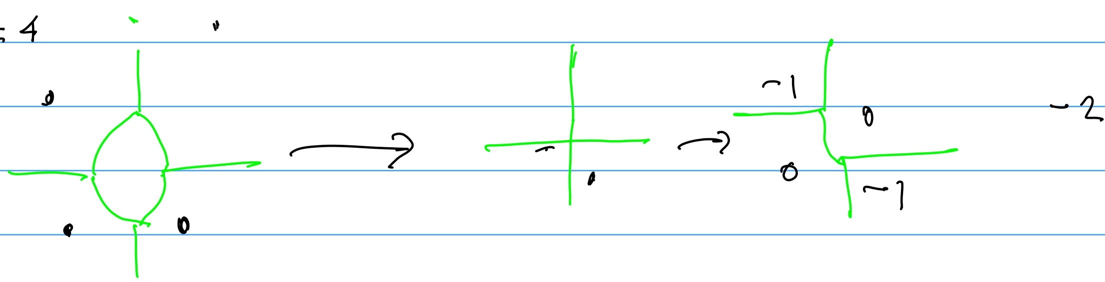
Deletion factor   cell deletion rate   what do A_{nj} look like?

τ relaxation time

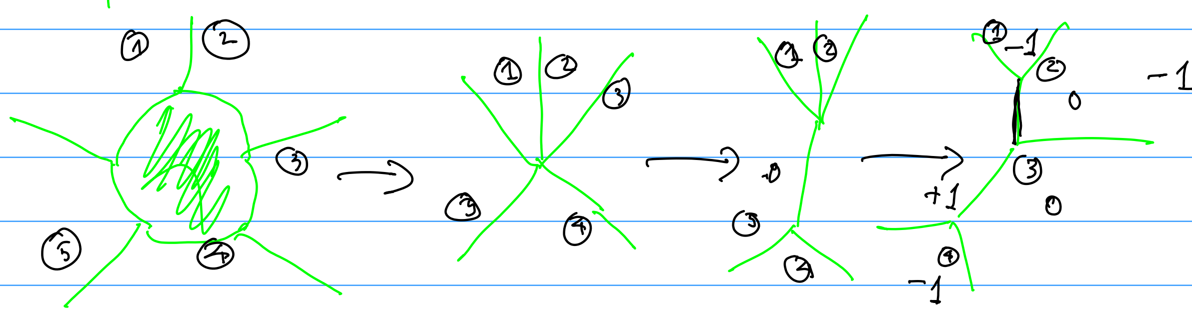
$n = 3$



$n = 4$



$n = 5$



μ sided cell deleted in time τ $\Rightarrow$ $6-\mu$ facets deleted in area $x = \alpha(6-\mu)\tau$

$$\sum_{3}^{5} (6-\mu) \int_0^{\alpha(6-\mu)\tau} f_\mu \, dx$$

cells deleted

---

facets deleted

---

all facet classes

$$\psi(t) = \lim_{\tau \to 0} \sum_{3}^{5} \frac{\mu-6}{\tau} \int_0^{\alpha(6-\mu)\tau} f_\mu \, dx \qquad \text{facet deletion rate}$$

$$= \alpha \sum (6-\mu)^2 f_\mu(0,t)$$

apportion itself over the facet classes:

$$\frac{\partial f_n}{\partial t} + \alpha(n-6)\frac{\partial f_n}{\partial x} = \psi(t) \cdot a_{n+1} \cdot \frac{f_{n+1}}{\int_0^\infty f_{n+1} \, dy} - \psi(t)\, a_n \frac{f_n}{\int_0^\infty f_n \, dy}$$

conditional prob cell loses a facet given it has $n+1$

probability a cell of $n+1$ facets has area x

+ contributions from facet interchange which we do not discuss

$a_n \geq 0$ constants $\quad a_3 + \cdots + a_{n_{max}} = 1$

harvest from large scale simulation

find they vary slowly

Would like to think

$$\frac{\partial f_n}{\partial t} + \alpha(n-6)\frac{\partial f_n}{\partial x} = \varepsilon_n \quad \text{small}$$

$f_n(x,0)$ "largest" for $n = 6$

likely not true

$$p(x,t) = \frac{\sum f_n(x,t)}{\sum \int_0^\infty f_n \, dy}$$