MD:

$x_i(t)$

$$E = \frac{1}{2}\sum_{i\neq j} \varphi(|x_i(t)-x_j(t)|)$$

$$m\ddot{x}_j(t) = f_j(t) = -\nabla_{x_j} E$$

MC:

$$H = -\sum_{\langle i,j\rangle} J_{ij}(\delta_{s_i s_j}-1) \qquad S_i = \pm 1$$

$$\bar{H} = \sum J_{ij}(S_i - S_j)^2 = H + \text{Const}$$

$S_i^2 = 1$

$$S_i - S_j \approx \nabla S$$

$$I = \int (S_i^2-1)^2 dx + \frac{1}{\varepsilon}\int J|\nabla S|^2 dx$$

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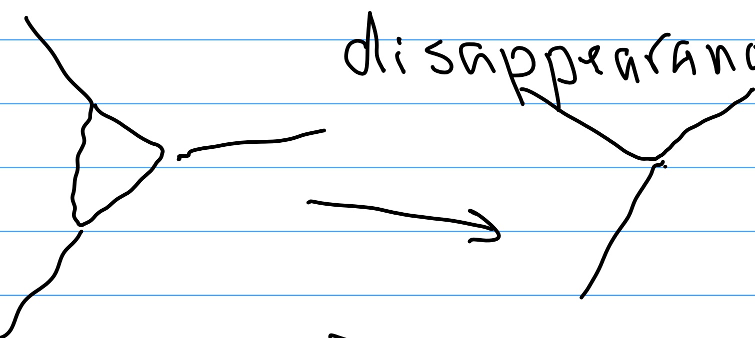
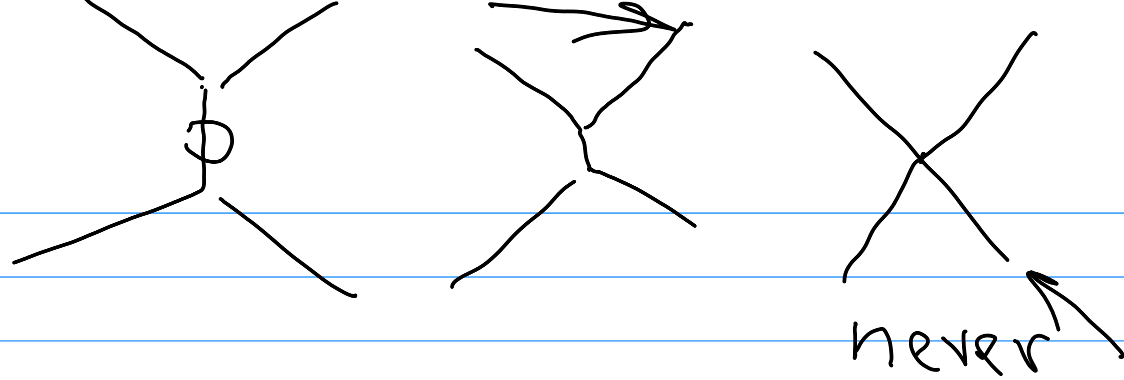
disappearana



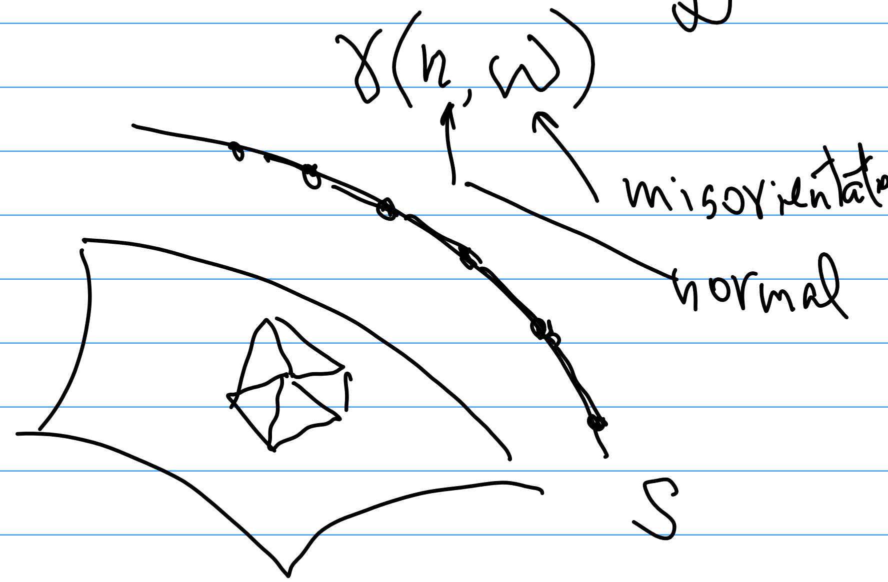
critical events



3D complicated.

Approx of PDE based models $\quad \gamma$ GB energy

approx S
triangulation



$$E = \sum_j \int_{\Gamma_j} \gamma(n, \omega)\, ds$$ total energy

(1) $v_n = -\mu \, \mathrm{div}_n (\nabla_n \gamma + \gamma n)$

$n \in S^2$

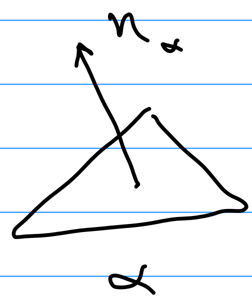
ω rotation matrix

5 dim.

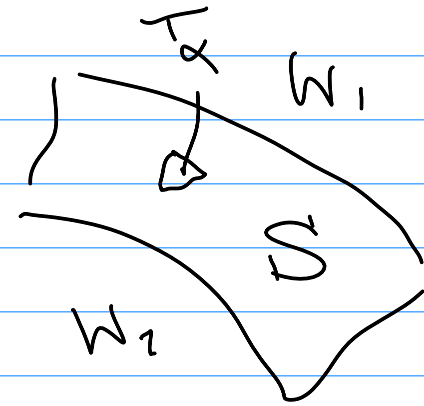
Discretization approx

② to ensure energy dissipation

best to approx E:



A_α area



$$E(t) = \sum_{\alpha \in T} \gamma(n_\alpha, \omega_\alpha) A_\alpha$$

T triangulation.

$x_j(t)$ point on S.

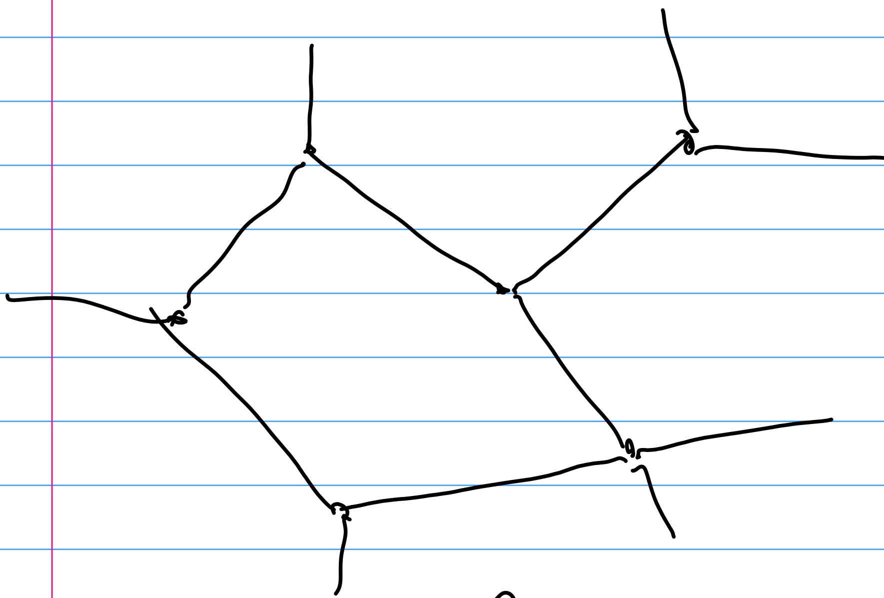
$$n_\alpha(t),\ A_\alpha(t) : \sum_\alpha \gamma(n_\alpha(t), \omega_\alpha) A_\alpha(t)$$

(2) $\dfrac{d}{dt} E(t) = \ldots$ discretize (1).

gradient flow in L_2 $\quad \int \frac{1}{\mu} v(x)^2 dx$

guarantees a good dissipation on discrete level !!

Vertex models 2D



all bd are straight.

Evolution?

$$E = \sum \gamma(n,\omega) \, |x_i(t) - x_j(t)|$$

$$\frac{dE}{dt} = \ldots \sum \langle \dot{X}_i , \cdot \rangle$$

norms: $\sum |\dot{X}_i|^2$