$$M.D.$$ 

$$X_i(t)$$

$$E = \frac{1}{2} \sum_{i \neq j} \Psi(|x_i(t) - x_j(t)|)$$

$$m \dot{x}_j(t) = f_j(t) = -\nabla_{x_j} E$$

$$M.C.$$ 

$$H = \sum_{i,j} J_{i,j} (\delta_{S_i, S_j} - 1) \quad S_i = \pm 1$$

$$\bar{H} = \sum J_{i,j} (S_i - S_j)^2 = H + \text{const}$$

$$S_i^2 = 1$$

$$S_i - S_j \approx \forall S$$

$$I = \int (S_i^2 - 1) dx + \int J \nabla S_i^2 dx$$

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disappearance

critical

events
3D complicated.

Approx of PDE based models of energy

$E = \sum_{j} \int_{s_{j}} \gamma(h, w) \, ds$

$\nabla \cdot \text{div} \left( \nabla \cdot \gamma + \gamma \eta \right) \quad \eta \in \mathbb{R}^2$

Discretization approx
To ensure energy dissipation, best to approx $E$:

$$E(t) = \sum_{\alpha \in \mathcal{T}} \chi(n_\alpha, \omega_\alpha) A_\alpha$$

$\mathcal{T}$: triangulation.

$n_\alpha(t), A_\alpha(t): \sum_\chi(n_\alpha(t), \omega_\alpha) A_\alpha(t)$

(2) \[ \frac{d}{dt} E(t) = \ldots \]

gradient  flow in $L^2$ \[ \int \frac{1}{\mu} \nabla u^2 dx \]

guarantees a good dissipation on discrete level!
Vertex models 2D

all bd are straight.

Evolution?

$$E = \sum \delta(n, \omega) |x_i(t) - x_j(t)|$$

$$\frac{dE}{dt} = \sum \langle \dot{x}_i, \cdot \rangle$$

norms: $$\sum |x_i|^2$$